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SENSITIVITY ANALYSIS OF DIVE PLANE REVERSAL OF SUBMERSIBLE VEHICLES AT LOW SPEEDS

by

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SENSITIVITY ANALYSIS OF DIVE PLANE REVERSAL OF SUBMERSIBLE VEHICLES AT LOW SPEEDS

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ABSTRACT

The capability of a submersible vehicle to accurately maintain its commanded depth, in a variety of operating speeds, depths, and loading conditions is critical for mission accomplishment. Below a certain critical speed, a phenomenon known as dive plane reversal occurs, where depth response changes sign with respect to a given dive plane command. This thesis builds on previous studies of the phenomenon and it presents a comprehensive study of dive plane reversal envelopes in the presence of external forces and moments on the vehicle. Based on these results, rational design and operational decisions can be made in order to avoid unpredictable vehicle response.

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I. INTRODUCTION

In submarine operations, one attribute critical to mission accomplishment is a submarine's ability to accurately maintain commanded depth. This is especially important when operating in shallow waters or at periscope depth when failure to maintain ordered depth could lead to easy detection of the sub or grounding.

At low speeds, submarines display a phenomenon commonly referred to as dive plane reversal. When operating at speeds sufficiently slow, dive plane commands must be reversed in order to achieve the desired behavior in depth change. The speed where this transition occurs is known as the critical speed.

Control systems for submersible vehicles must have the ability to recognize when the vehicle is operating at about or below the critical speed. Failure to do so could lead to erratic behavior of the vehicle. This is especially important when designing the control system for an unmanned submersible, which operates with little or no human intervention. In this case, recognition of the critical speed as well as dive commands once below this threshold must all take place automatically.

Using previous studies on the phenomenon of dive plane reversal [Refs 1&2], this thesis presents a comprehensive sensitivity study of dive plane reversal envelopes under the influence of a variety of external forces and moments on the vehicle. Submersible control systems utilizing the results displayed in this thesis will be better equipped to maintain ordered depth at low speeds and therefore avoid unpredictable vehicle response.

II. VEHICLE MODELING

A. INTRODUCTION

For the purposes of this study in relation to the maneuvering and kinematic analysis of the vehicle, the following assumptions apply:

1) The vehicle behaves as a rigid body.

2) In regards to the acceleration of the center of mass of the vehicle, the Earth's rotation is negligible.

3) The primary forces that act on the vehicle are inertia, gravity, hydrostatics and hydrodynamics.

B. COORDINATE SYSTEMS AND POSITIONAL DEFINITIONS

In order to study the motion of a submersible vehicle a system for describing its position must first be defined. A global coordinate frame, *OXYZ*, is defined with origin *O*, and a set of axes aligned with directions North, East and Down. Aligned with this is a right-hand reference frame with unit vectors $\bar{I}$, $\bar{J}$ and $\bar{K}$. The necessity that the $\bar{K}$ direction is chosen as positive downwards results from the convention of referencing a submarine's depth downwards from the water's surface. Neglecting the earth's rotation, the *IJK* coordinate frame becomes an inertial reference frame in which Newton's Laws of Motion will be valid. A vehicle's position, $\bar{R}_0$, in this frame will have the vector components, $\bar{R}_0 = [X_0\bar{I} + Y_0\bar{J} + Z_0\bar{K}]$. From the perspective of the vehicle, a standard convention will be used that places the Y axis to the right while facing along the X axis, and the Z axis is positive downwards.

A body centered coordinate frame O'_{xyz}, is defined with the origin O', located on the centerline of the vehicle, translating and rotating with the vehicle. The vehicle's center of mass may or may not be concurrent with the origin O'. This origin will be the

point about which all vehicle body forces will be computed. By convention, the body centered coordinate frame will have its origin O' at the ship's center, with unit vectors, $\bar{i}$, $\bar{j}$ and $\bar{k}$. The x axis is defined parallel to the ship's longitudinal axis with the z axis vertically down. Generally, the ship's center of gravity and center of buoyancy are not located at the body centered origin, nor are they collocated. These points are defined by C_G and C_B respectively. The locations of the center of buoyancy and center of mass have considerable bearing on this study. In Newtonian physics, the rate of change of linear momentum of the center of mass and moments about the body center of mass is equal to the rate of change of angular momentum. This is significant as the center of buoyancy of a submerged body is defined by its shape, while its center of gravity is determined by the distribution of weight over the body.

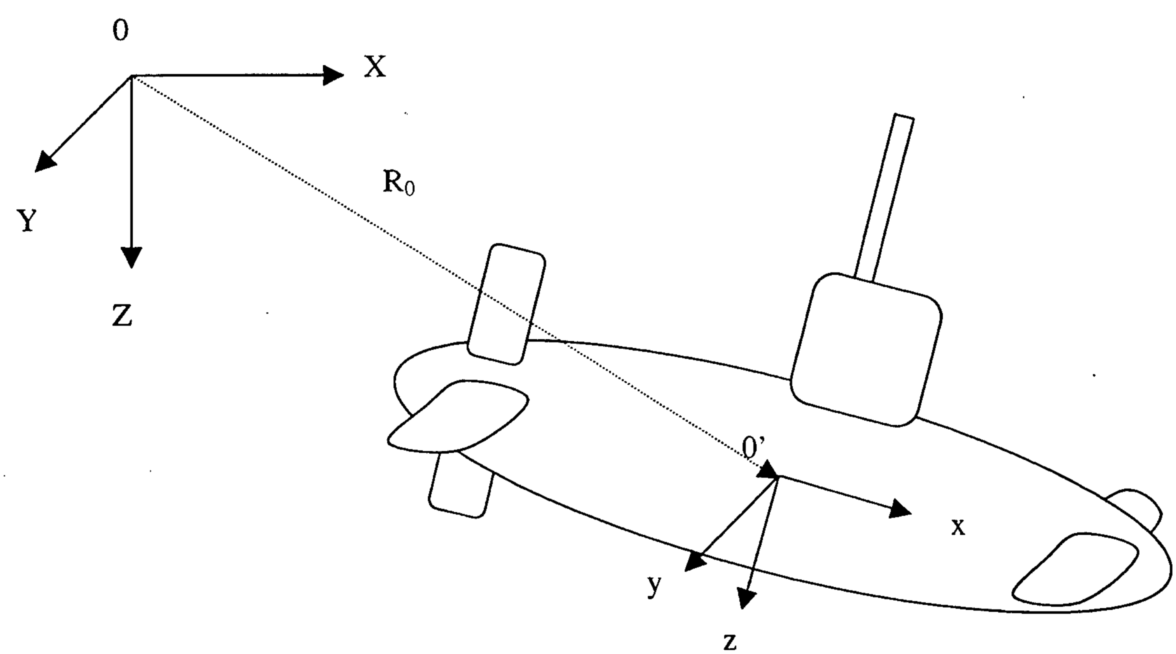

Figure 2-1 Coordinate Axes Representation

C. ANGULAR POSITION IN THE GLOBAL COORDINATE FRAME

For the purposes of this thesis, the attitude of the vehicle is described by three angles relative to the global coordinate frame. The three terms used to describe attitude are azimuth, elevation and spin. Azimuth is the direction in the horizontal plane that the vehicle points. Elevation is the angle above or below the horizontal plane that the vehicle points. Finally, roll is the angle that the vehicle is rotated about its own longitudinal axis.

It should be noted that when a vehicle approaches a purely vertical angle of attack (ie an elevation angle of 90 degrees), spin angle cannot be differentiated from azimuth angle. For computer simulations and control systems, a more complicated system to describe angular position is usually required. However, for the purposes of this study, azimuth, elevation and spin are sufficient to describe a vehicle's attitude.

D. ROTATIONAL TRANSFORMATIONS

When describing positional, velocity and acceleration vectors, it is frequently more useful to describe them in relation to a body rather than that of the global coordinate frame. To facilitate this, a transformation system must be defined to relate global coordinates to body centered coordinates. The forward transformation starts with the global coordinate frame. Three rotations are performed to align a frame parallel to the body centered coordinate frame. Azimuth rotation, ψ, is defined as a positive rotation about the global Z axis. Elevation, θ, is defined as positive rotation about the new Y axis. Finally, roll, ϕ, is defined as positive rotation about the new X axis. These three rotations uniquely define the angular orientation of the body.

Consider a position vector, $\overline{R}_0$, defined in the global coordinate frame by $\overline{R}_0 = [x_0, y_0, z_0]'$. It can be seen that $\overline{R}_0$ will have different coordinates when related to a body centered coordinate system that has been rotated by an azimuth angle of ψ about the global Z axis.

Referring to Figure 2.2, the position vector, $\overline{R}_1$, is related to the global coordinate frame using the following relationships:

$$x_1 = x_0 \cos\psi + y_0 \sin\psi \tag{2.1}$$

$$y_1 = -x_0 \sin\psi + y_0 \cos\psi \tag{2.2}$$

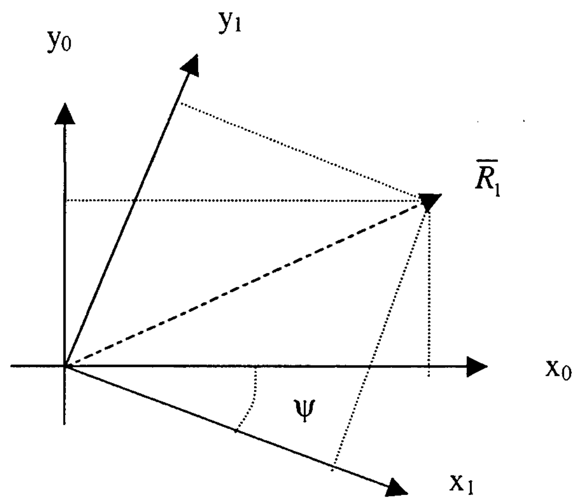

Figure 2-2 Azimuth Rotation

These relationships can be represented in matrix form using the rotation matrix operation,

$$\overline{R}_1 = [T_{\psi z}]\ \overline{R}_0; \tag{2.3}$$

where $[T_{\psi z}]$ is an orthogonal transformation[Ref 1]. Premultiplying any vector, $\overline{R}_0$, by the transformation $[T_{\psi z}]$, yields the components of the vector in the rotated coordinate frame. Extending this rotation to three directions yields a combined total rotation transformation,

$$T(\theta,\phi,\psi) = T(\theta)T(\phi)T(\psi). \tag{2.4}$$

Expanding equation 2.4 results in the following matrix:

$$\begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi & \cos\theta\sin\phi \\ \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi & \cos\theta\cos\phi \end{bmatrix}$$

Using the above transformation, any vector in the global coordinate frame can be expressed in coordinates of a rotated reference frame using

$$\overline{R}_{rotated} = [T(\theta,\phi,\psi)]\,\overline{R}_{global}\,. \tag{2.5}$$

E. KINEMATICS

In order to study the motion of an object, definitions of translational and rotational velocities must be established. These definitions are utilized when further studying the effect of forces and accelerations on the object. Translational velocity is defined as,

$$\dot{\overline{R}} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} \tag{2.6}$$

Using the rotational transformations described in the previous section, this global velocity vector can be translated into a body fixed reference frame. Let [u, v, w]' be the body fixed components of the velocity vector, where u is the vehicles forward speed, v is its side slip velocity (sway) and w is its speed in relation to its local vertical axis (heave). The body fixed velocity can be determined via the forward transformation:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = T(\phi,\theta,\psi)\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} \tag{2.7}$$

Transformation back to global coordinates is accomplished by the reverse transformation:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = T^{-1}(\phi,\theta,\psi) \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{2.8}$$

Using these relations, the global motion of a vehicle can be described in terms of its angular attitude and its body centered velocity components.

Transformation of rotational velocities from global coordinates to body centered coordinates is more complicated. Because elevation angle, θ, is made about an intermediate y axis based upon azimuth and spin angle, ϕ, is made about an intermediate x axis based upon the elevation angle, the transformation is a three step process. Let ω be the rotation rate of the body relative to its own body aligned reference frame and p, q and r be the azimuth, elevation and spin components of ω. ω is calculated from global angular velocities via the following equation:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \Sigma T(\phi)T(\theta)T(\psi) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + T(\phi)T(\theta) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + T(\phi) \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} \tag{2.9}$$

Simplifying equation 2.9 yields:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} -\dot{\psi}\sin\theta + \dot{\phi} \\ \dot{\psi}\sin\theta + \dot{\theta}\cos\phi \\ \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \end{bmatrix}. \tag{2.10}$$

Calculating global angular velocities from body centered angular velocities is accomplished by inverting equation 2.10:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ q\cos\phi - r\sin\phi \\ (q\sin\phi + r\cos\phi)/\cos\theta \end{bmatrix}. \tag{2.11}$$

F. EQUATIONS OF MOTION

Newtonian physics is the basis for describing motion of an object and relative forces involved. In order to apply $\overline{F} = m\overline{A}$ to translational motion, acceleration of an object within a rotating reference frame must first be defined. This acceleration is derived by differentiating the position vector $\overline{R}_0$ with respect to time twice.

$$\ddot{\overline{R}}_G = \dot{\overline{v}} + \dot{\overline{\omega}} \times \overline{\rho}_G + \overline{\omega} \times \overline{\omega} \times \overline{\rho}_G + \overline{\omega} \times \overline{v}. \tag{2.12}$$

Forces applied to an object are equated to the product of this acceleration and mass of the object.

$$\overline{F} = m\left\{\dot{\overline{v}} + \dot{\overline{\omega}} \times \overline{\rho}_G + \overline{\omega} \times \overline{\omega} \times \overline{\rho}_G + \overline{\omega} \times \overline{\rho}_G\right\} \tag{2.13}$$

For a submersible vehicle, this force vector is comprised of gravitational, hydrostatic and hydrodynamic forces acting on the vehicle.

For rotational equations of motion the rate of change of angular momentum about an object's center of mass is equated by the sum of moments applied about the center of mass. The mass moments of inertia for an object are defined as follows:

$$I_0 = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}. \tag{2.14}$$

Angular momentum for an object is defined as follows:

$$\overline{H}_0 = I_0\overline{\omega}. \tag{2.15}$$

The rate of change of angular momentum is defined as follows:

$$\overline{M}_0 = \dot{\overline{H}}_0 + \overline{\rho}_G \times \left(m\ddot{\overline{R}}_G\right) \tag{2.16}$$

Using equations 2.12 and 2.16, the total applied moments on an object about the origin is given as follows:

$$\dot{\overline{H}}_0 = I_0\dot{\overline{\omega}} + \overline{\omega} \times \overline{H}_0. \tag{2.17}$$

where the acceleration of the global position vector is given by,

$$\ddot{\overline{R}}_0 = \dot{\overline{v}} + \overline{\omega} \times \overline{v}. \tag{2.18}$$

Combining these elements, the rotational equation of motion becomes:

$$\overline{M}_0 = I_0\dot{\overline{\omega}} + \overline{\omega} \times \left(I_0\overline{\omega}\right) + m\left\{\overline{\rho}_G \times \dot{\overline{v}} + \overline{\rho}_G \times \overline{\omega} \times \overline{v}\right\} \tag{2.19}$$

G. VERTICAL FORCES

The two vertical forces always present in submersible dynamics are submersible weight and buoyancy. For a submersible to be neutrally buoyant, the magnitudes of its weight and buoyancy must be equal. A submersible's weight is defined by its mass and acts through the center of gravity of the submersible. A submersible's buoyancy is defined by its shape and acts through the submersible's center of buoyancy. Usually, but not always, the centers of buoyancy and gravity are located on the same vertical line of action. For a submersible to be statically stable, the center of buoyancy must be located above the center of gravity. As such there is usually a vertical separation between the centers of buoyancy and gravity.

Assume that a submersible is operating at a constant depth at a pitch angle $\theta = 0$ and roll angle $\phi = 0$ and that its centers of buoyancy and gravity are act through the same vertical line of action. Any change in pitch or roll will cause a lateral separation of the respective lines of action of buoyancy and gravity. As a result, a moment will develop acting to force the submersible back to its original orientation. The resolved vertical force based upon weight, buoyancy, pitch and roll is,

$$\bar{f}_g = (W - B)\begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} \tag{2.20}$$

and always acts in the global vertical direction. Note that if the magnitudes of weight and buoyancy are equal, this resolved force is zero. The resolved moment based upon weight, buoyancy, pitch and roll is,

$$\overline{m}_g = W\overline{\rho}_G \times \begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} - B\overline{\rho}_B \times \begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix}. \tag{2.21}$$

Because there is a vertical separation between the centers of gravity and buoyancy, a moment will develop with changes in pitch and roll even if the magnitudes of weight and buoyancy are equal.

III. DIVE PLANE REVERSAL

In order to study the phenomenon of dive plane reversal, dive plane function must first be understood. There are several ways to control the depth of a submersible vehicle. Buoyancy can be altered or adjustments can be made to either the fairwater or dive planes. The primary method of depth control is the utilization of dive planes.

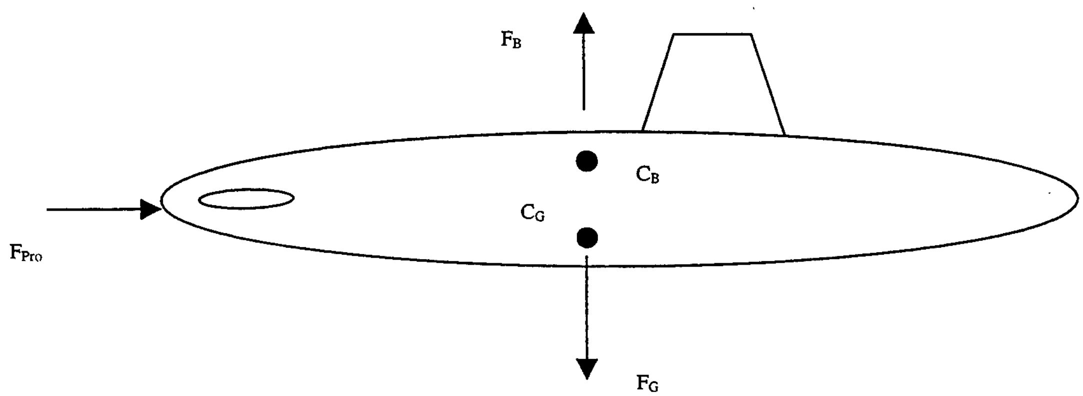

Figure 3-1 Submersible's initial configuration

Assume a submersible vehicle is travelling through the water with a speed $\bar{v}$ at a constant depth with a pitch angle of $\theta = 0$. Referring to Figure 3-1, the forces acting on the submersible are its buoyancy, gravity and its propulsion force. To descend, a submersible's dive planes are deflected to an upward angle of attack α. An upward force, $\overline{F}_{plane}$ which is a function of α and v^2 results on the dive planes. Acting through the submersible's metacenter, $\overline{F}_{plane}$ creates a moment, $\overline{M}_{plane}$ which acts to pitch the submersible forward to some negative pitch θ. Referring to Figure 3-2, as the submersible pitches forward, the centers of buoyancy and gravity which were vertically aligned originally achieve a small amount of longitudinal separation and a couple develops. This moment, $\overline{M}_{pitch}$ acts to oppose the action of $\overline{M}_{plane}$ and is strictly a function of the submersible's pitch angle θ. Eventually, as the pitch angle increases, the magnitudes of $\overline{M}_{pitch}$ and $\overline{M}_{plane}$ will equalize and the submersible will stabilize at that pitch angle.

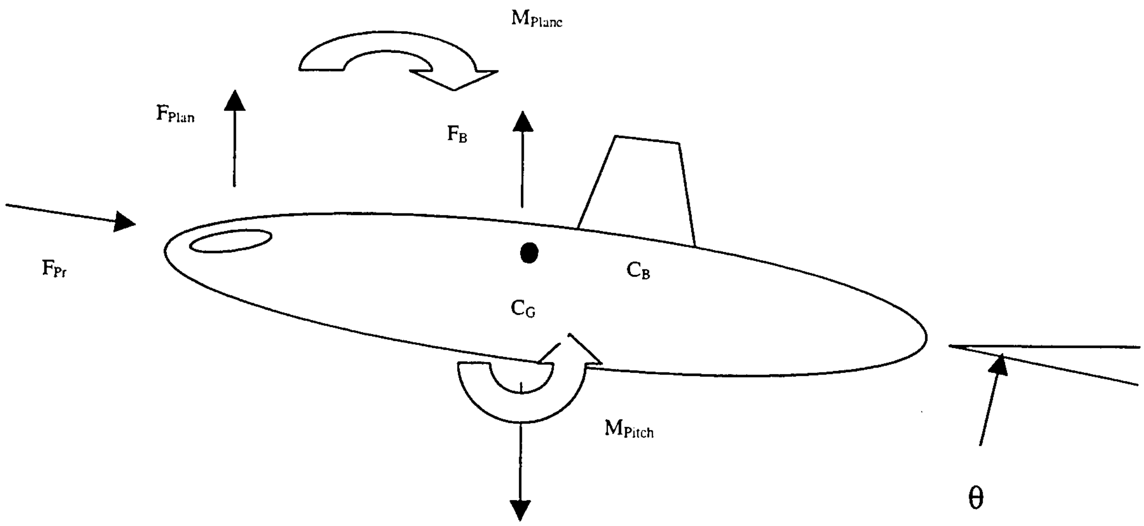

Figure 3-2 Submersible's Diving Configuration

Dive plane reversal is a phenomenon that occurs below a speed known as the critical speed. Below the critical speed the hydrodynamic force $\overline{F}_{plane}$ and subsequent moment $\overline{M}_{plane}$ aren't large enough appreciably alter the pitch of the submersible. Consequently, the only force acting to change the depth of the submersible is $\overline{F}_{plane}$. A submersible attempting to dive using its dive planes below the critical speed will actually rise towards the surface.

Onboard the Navy's operational submarine fleet, dive plane reversal is well understood. When operating below a submarine's critical speed, its operators know to take different actions to control depth. However, with unmanned submersible vehicles, the phenomenon of dive plane reversal leads to complicated problems in the area of automated control systems. An unmanned submersible's control system must be able to recognize when the submersible is operating below the critical speed. Additionally, it must take appropriate action to control and maintain depth. Without these important considerations, control actions could lead to erroneous results and instabilities in vehicle control. For a vehicle operating in restricted waters, shallow depths and engaged in

delicate activities such as mine hunting or sweepings, the importance of a control system to be stable below the critical speed increases drastically.

IV. RESEARCH

Under certain conditions, submersible vehicles can be influenced by external forces and moments due to operating conditions. Surface effects are vertical forces resulting from a pressure differential as a result of operating near the surface. Additionally, external moments may be produced due to surface effects depending upon the angle the vehicle's longitudinal axis makes with the surface. Similarly, bottom effects are vertical forces resulting from a pressure differential as a result of operating near the ocean floor.

The focus of this thesis is to explore the effects that external forces and moments have on a submersible's critical speed. This research utilized the MATLAB program forcer.m. This program numerically calculates the critical speed envelopes of the DARPA SUBOFF model based on a set of input characteristics. The DARPA SUBOFF model is used in this simulation as a generic body of revolution for which a set of hydrodynamic coefficients and geometric properties is readily available. The program forcer.m is a modified version of bifsu.m [Ref 1] and was modified to utilize user defined inputs of external vertical forces and moments. During simulation the effect of these external forces and moments can be observed.

For this study the magnitudes of weight and buoyancy were assumed to be equal. Using body centered coordinate frame aligned with the submersible's geometric center Newton's equations of motion are expressed as:

$$
\begin{aligned}
m(\dot{w} - Uq - z_G q^2 - x_G \dot{q}) &= Z_{\dot{q}}\dot{q} + Z_{\dot{w}}\dot{w} + Z_q Uq + Z_w Uw \\
&- C_D \int_{stern}^{bow} b(x) \frac{(w - xq)^3}{|w - xq|} dx + F_Z \cos\theta + U^2 Z_\delta \delta
\end{aligned}
\tag{4.1}
$$

$$
\begin{aligned}
I_y \dot{q} + m z_G wq - m x_G(\dot{w} - Uq) &= M_{\dot{q}}\dot{q} + M_{\dot{w}}\dot{w} + M_q Uq + M_w Uw + x_F F_z \cos\theta \\
-(z_G W - z_G B)\sin\theta + U^2 M_\delta \delta &- C_D \int_{stern}^{bow} b(x) \frac{(w - xq)^3}{|w - xq|} x dx - (x_G W - x_B B)\cos\theta
\end{aligned}
\tag{4.2}
$$

In equations (4.1) and (4.2), m is the vehicle's mass, U is the vehicles forward speed, q is the pitch rate or $\dot{\theta}$, Z is the heave force, w is the heave velocity I_y is the vehicle's mass moment of inertia and δ denotes the stern plane deflection. The coordinates for the vehicle's centers of gravity and buoyancy are denoted by (x_G ,z_G) and (x_B ,z_B) respectively. *b(x)* denotes the beam of the vehicle as a function of length. To simplify analysis, coefficient of drag, C_D is assumed to be constant over the length of the vehicle [Ref. 1]. The external vertical force and point of application are expressed by F_z and x_F. Subscripts represent the derivative of the variable with respect to the subscripted value. For example, M_w represents the derivative of pitch moment with respect to heave velocity.

Using these conventions, the rate of change of depth is calculated by:

$$\dot{z} = -U \sin\theta + w\cos\theta\ . \tag{4.3}$$

To simplify analysis, the forward velocity of the vehicle U is assumed to remain constant. Assuming steady state, equations (4.1), (4.2) and (4.3) algebraically reduce to:

$$\begin{aligned}&(Z_w M_\delta - M_w Z_\delta)Uw - C_D A_w (M_\delta - x_A Z_\delta) w|w| + [F_z M_\delta \\ &+ (x_G W - x_B B + F_z x_F) Z_\delta]\cos\theta + Z_\delta (z_G W - z_B B)\sin\theta = 0\end{aligned} \tag{4.4}$$

where,

$$\sin\theta = \frac{w}{\sqrt{w^2 + U^2}}\ , \qquad \cos\theta = \frac{U}{\sqrt{w^2 + U^2}}$$

$$A_w = \int b(x)dx\ , \qquad x_A = \frac{1}{A_w}\int xb(x)dx$$

The effects of control surface saturation are not taken into account in equation (4.4). As a result, steady state solutions in pitch are allowed to achieve unrealistically high values. In order to account for control surface saturation, $\delta = \delta_{sat}$ is utilized, where

$\delta_{sat} = \pm 0.4$ radians. As a result, $w \neq U \tan\theta$. Steady state values are calculated from a heave velocity equation:

$$Z_w U w - C_D A_w w|w| - F_z \cos\theta + Z_\delta U^2 \delta_{sat} = 0 \tag{4.5}$$

and a pitch rate equation:

$$\begin{aligned} &M_w U w - C_D x_A A_w U^2 w|w| - (x_G W - x_B B - x_F F_z)\cos\theta \\ &-(z_G W - z_B B)\sin\theta + M_\delta U^2 \delta_{sat} = 0 \end{aligned} \tag{4.6}$$

which are utilized in the calculation.

During calculation, forcer.m identifies a value of Xgb/L corresponding to the critical speed based on loading conditions. Xgb/L is simply a nondimensional parameter describing the longitudinal separation of the submersible's center's of gravity and buoyancy divided by its overall length. Critical speed is converted to a Froude number defined as,

$$Fn = U / \sqrt{g z_{GB}}\,. \tag{4.7}$$

Calculations are performed over a range of speeds. Critical Froude number is subsequently defined as,

$$Fn_c = \left[\frac{Z_\delta W}{(M_w Z_\delta - Z_w M_\delta) g} \right]^{1/2}. \tag{4.8}$$

Although Froude number is strictly associated with surface vessels and does not normally apply to fully submerged bodies, it is applicable here. Froude number is a nondimensional term describing the ratio of hydrodynamic and hydrostatic forces. In the case of equation (4.4), that is precisely what it is quantifying. A submersible's critical speed will depend primarily on its metacentric height. Another advantage of using Froude number is that because it is a nondimensional term, it can be appropriately scaled for use with different submersibles. After the program has performed its calculations, it plots critical Froude number as a function of Xgb/L.

To research the affect of external forces and moments on critical speed, forcer.m is run repeatedly. By varying force magnitude, force direction, point of application coefficient of drag, conclusions can be made as to the influence these parameters have on the determination of critical speed.

V. RESULTS

The simulation was originally run with no external forces and moments present to produce a baseline plot of the critical speed envelopes. Figure 5-1 displays these envelopes at three different coefficients of drag. Referring to Figure 5-1, the peaks of the plots are all centered at an Xgb/L value of 0. As the longitudinal separation of the centers of buoyancy and mass increases the corresponding critical speed decreases. Similarly, as the coefficient of drag increases, the critical speed decreases.

Figure 5-2 displays the critical speed envelopes in the presence of a upwards vertical force at a coefficient of drag of 0.0. These forces are applied at the submersible's metacenter and hence incur no moment on the submersible. Referring to Figure 5-2, increasing in force magnitude yields a shift of the critical speed curve to a positive value of Xgb/L as well as a slight decrease in the peak value of critical speed. The relative magnitude of these forces to the submersible's overall weight is very small (< 0.5%). Yet, even at these small magnitudes, significant shifts in the critical speed envelopes result. Similar trends are displayed on Figures 5-3 and 5-4 for coefficients of drag of 0.1 and 0.3 respectively.

Figure 5-5 displays the critical speed envelopes in the presence of a downwards vertical force. Similarly as before with upwards forces, there is a decrease in critical speed with increasing applied force. However, with negative vertical forces present, the shift of the critical speed curve is to a negative value of Xgb/L. Similar trends are displayed on Figures 5-6 and 5-7 for coefficients of drag of 0.1 and 0.3 respectively.

Figure 5-8 displays the critical speed envelopes in the presence of a positive external moments at a C_D of 0.0. For comparison, these moments were produced by forces equal in magnitude as in previous plots but applied at a point a quarter of the submersible's length along its longitudinal axis. Again, a trend of decreasing critical speed is observed in the presence of greater external force. However, the Xgb/L shift is less pronounced when comparing external moments to external force. Similar trends are displayed on Figures 5-9 and 5-10 for coefficients of drag of 0.1 and 0.3 respectively.

Figure 5-11 displays the critical speed envelopes in the presence of a positive external moments at a C_D of 0.0. Predictably, critical speed decreases as the magnitude of the moment is increased. The Xgb/L shift is negative and not as pronounced as that in the presence of external negative vertical force. Similar trends are displayed on Figures 5-12 and 5-13 for coefficients of drag of 0.1 and 0.3 respectively.

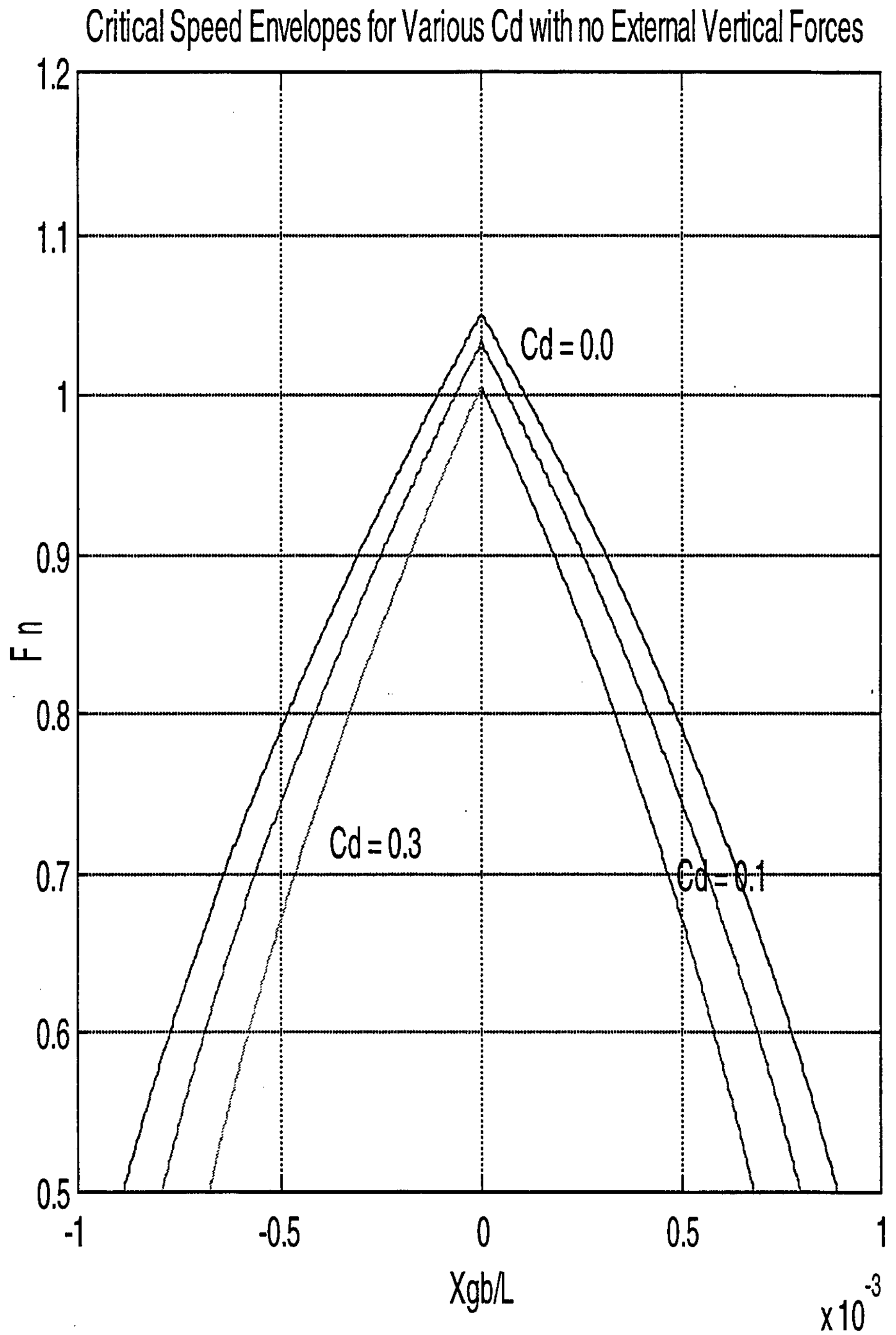

Figure 5-1 Baseline Critical Speed Envelopes

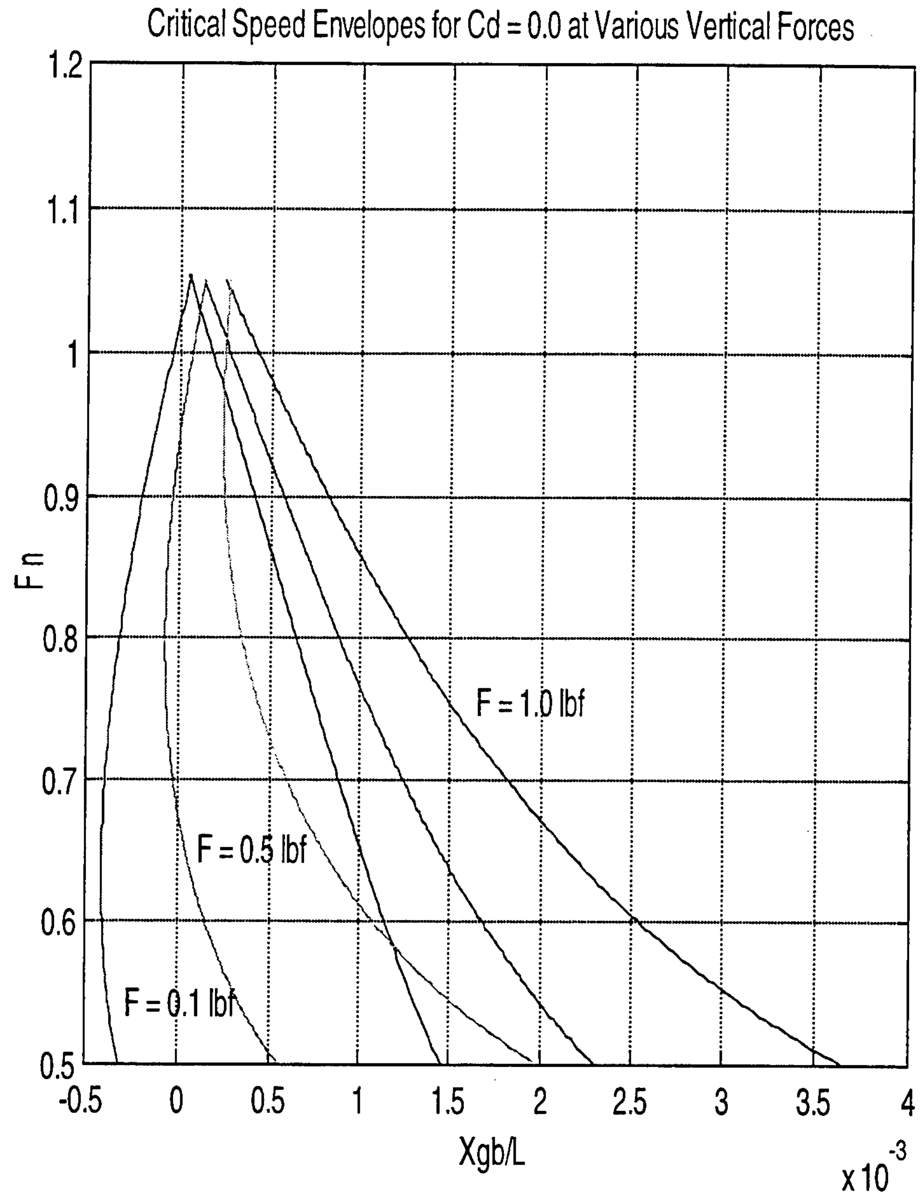

Figure 5-2 Critical Speed Envelopes Subject to Various Positive Vertical Forces at C_D = 0.0

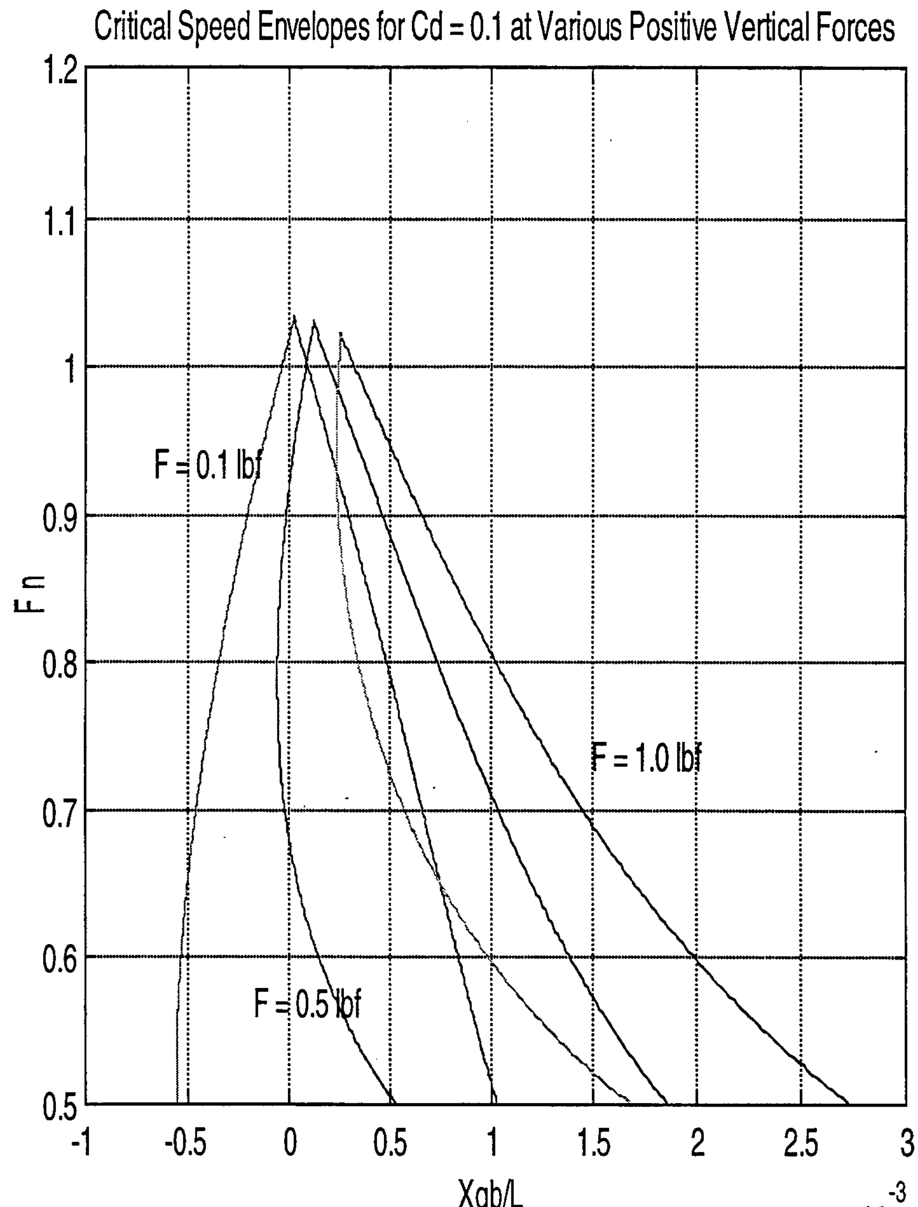

Figure 5-3 Critical Speed Envelopes Subject to Various Positive Vertical Forces at $C_D = 0.1$

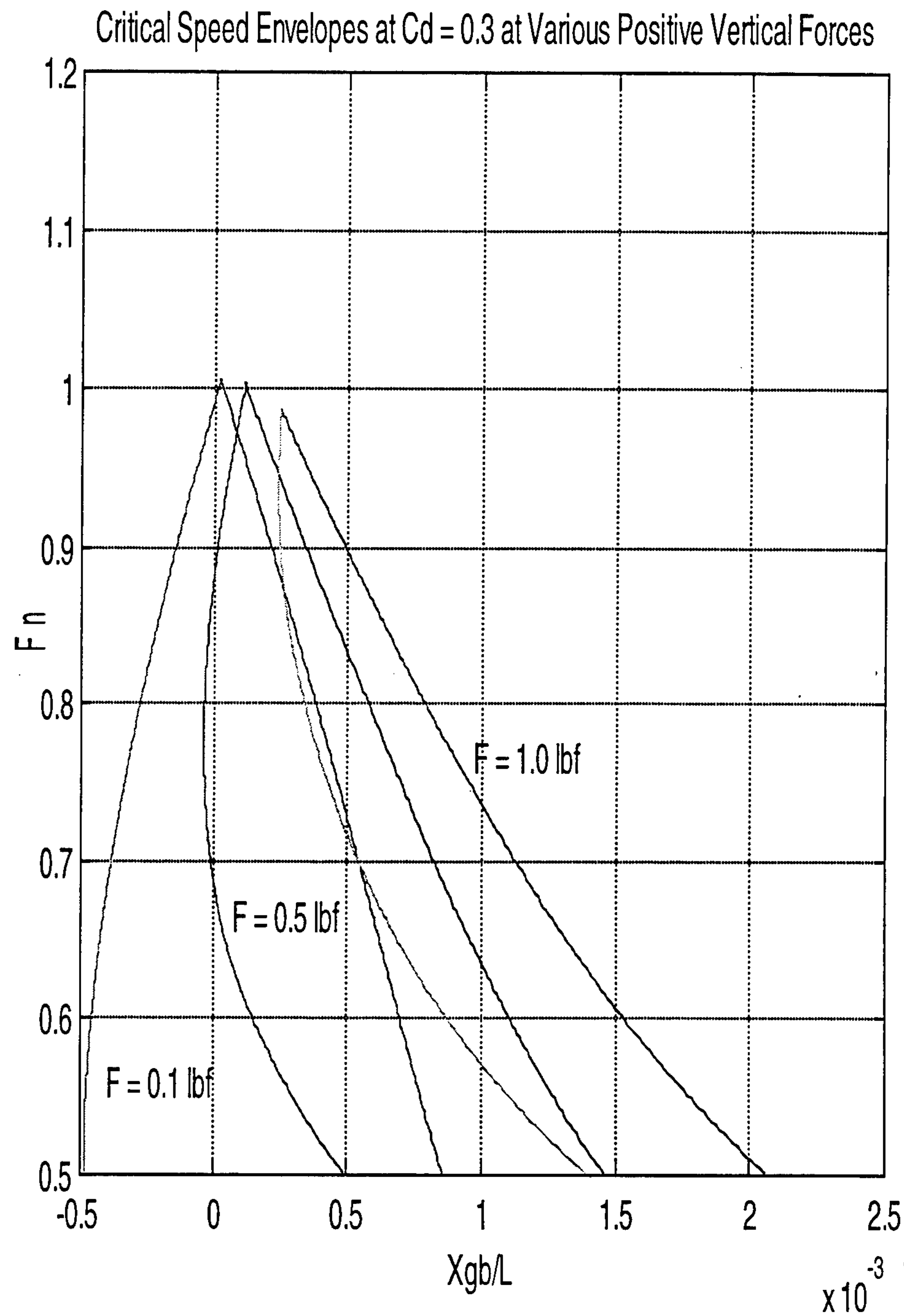

Figure 5-4 Critical Speed Envelopes Subject to Various Positive Vertical Forces at $C_D = 0.3$

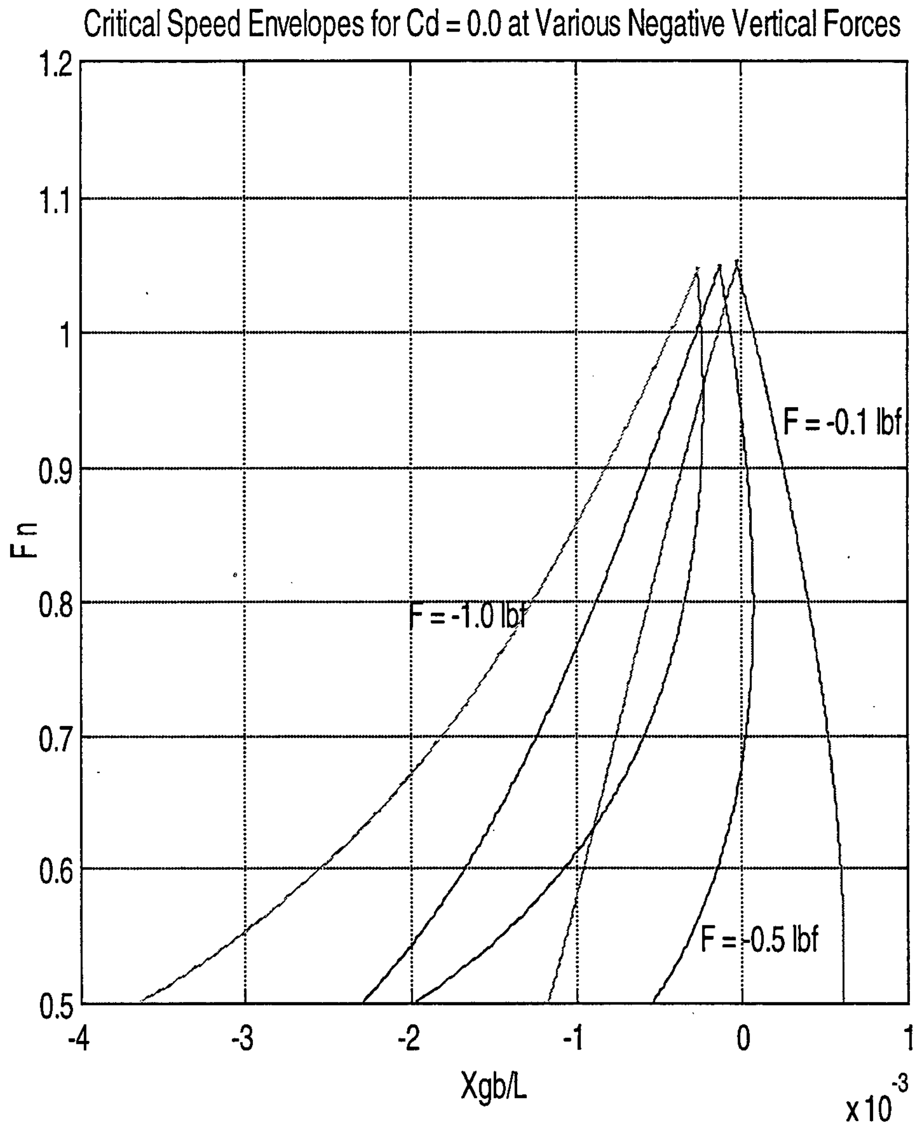

Figure 5-5 Critical Speed Envelopes Subject to Various Negative Vertical Forces at $C_D = 0.0$

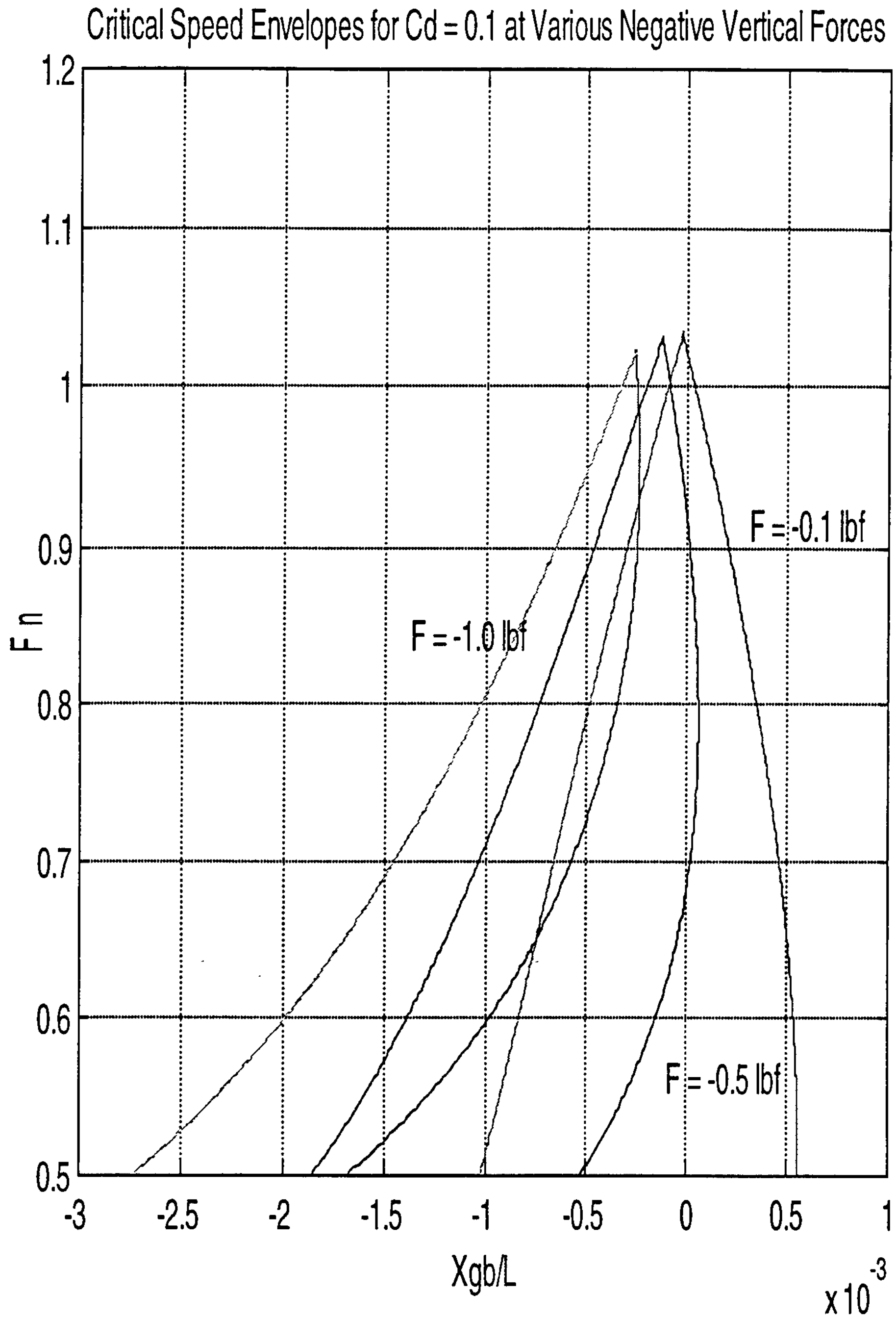

Figure 5-6 Critical Speed Envelopes Subject to Various Negative Vertical Forces at $C_D = 0.1$

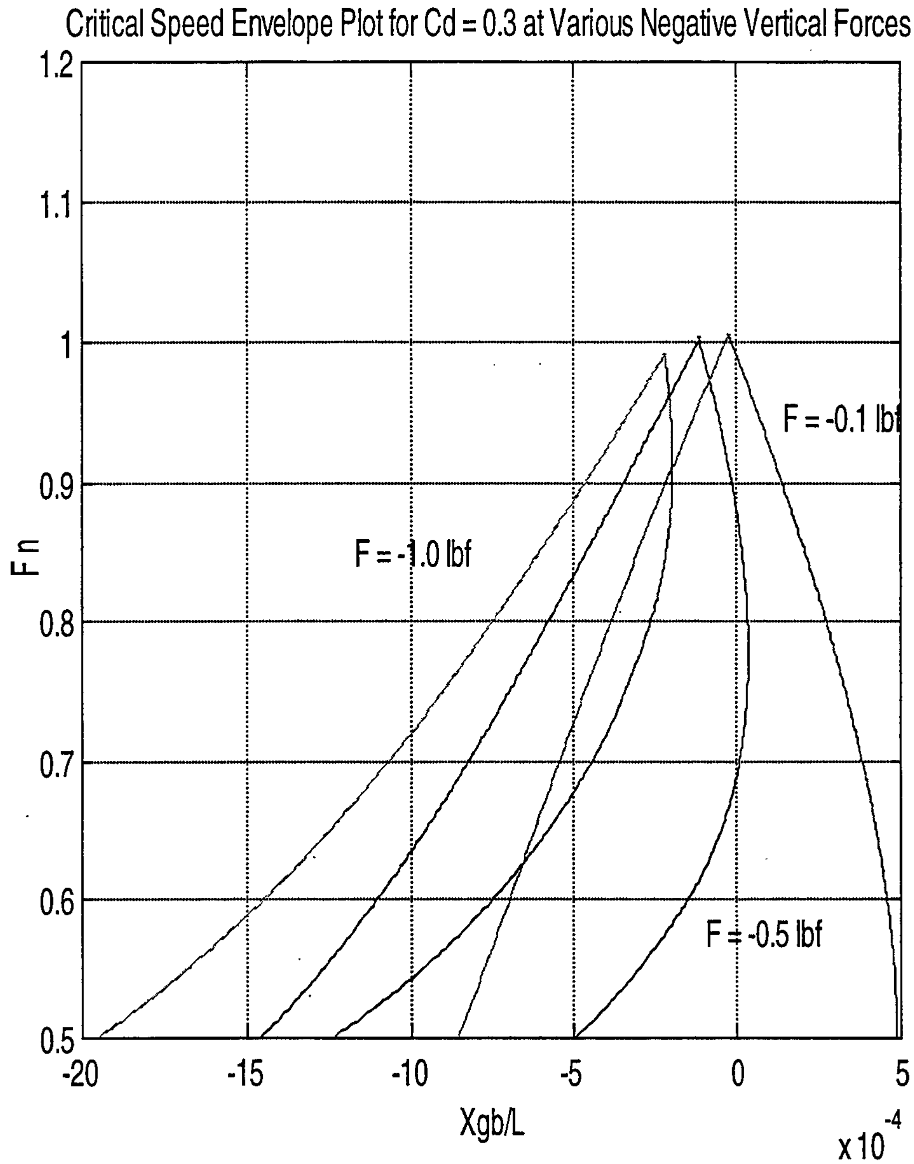

Figure 5-7 Critical Speed Envelopes Subject to Various Negative Vertical Forces at $C_D = 0.3$

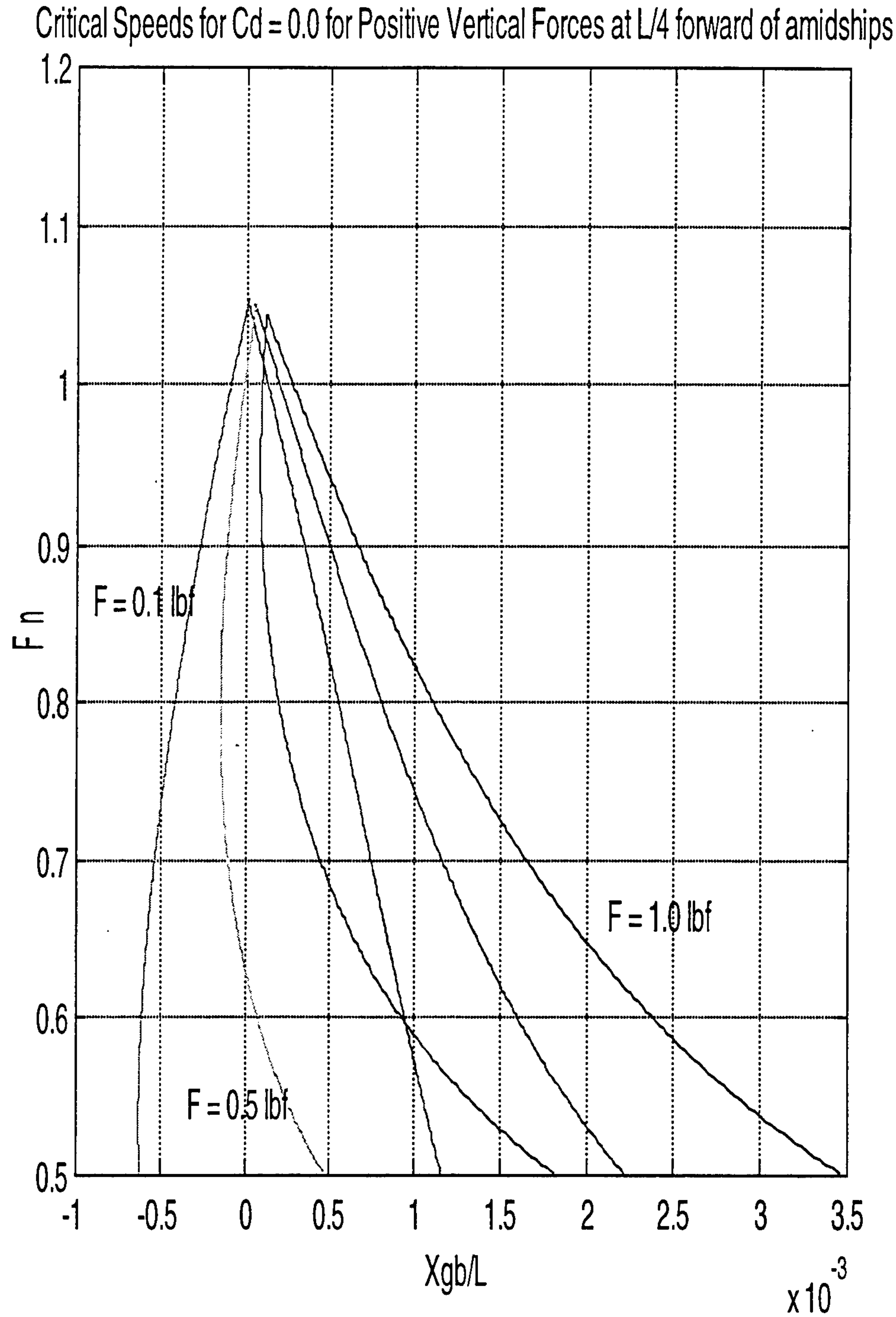

Figure 5-8 Critical Speed Envelopes Subject to Various Positive Moments at $C_D = 0.0$

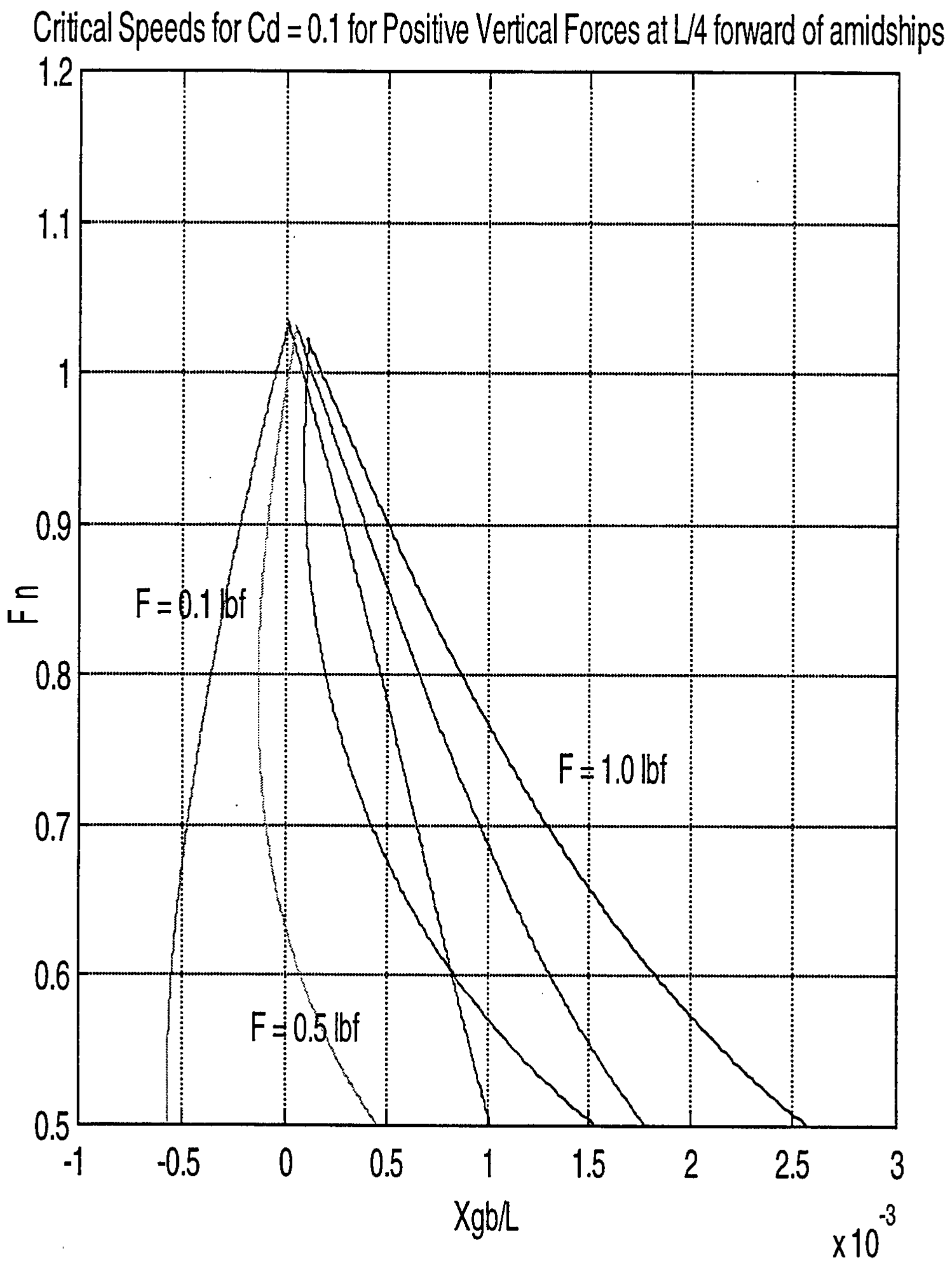

Figure 5-9 Critical Speed Envelopes Subject to Various Positive Forces at $C_D = 0.1$

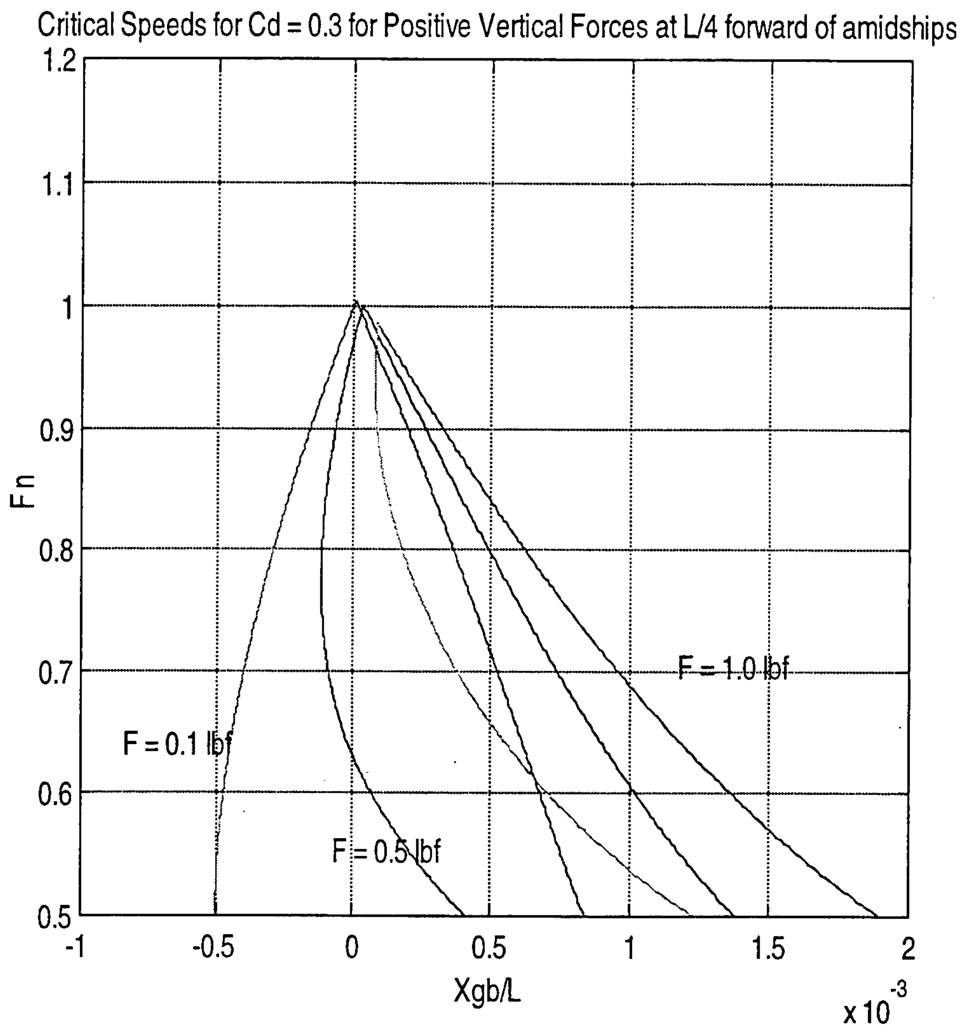

Figure 5-10 Critical Speed Envelopes Subject to Various Positive Moments at $C_D = 0.3$

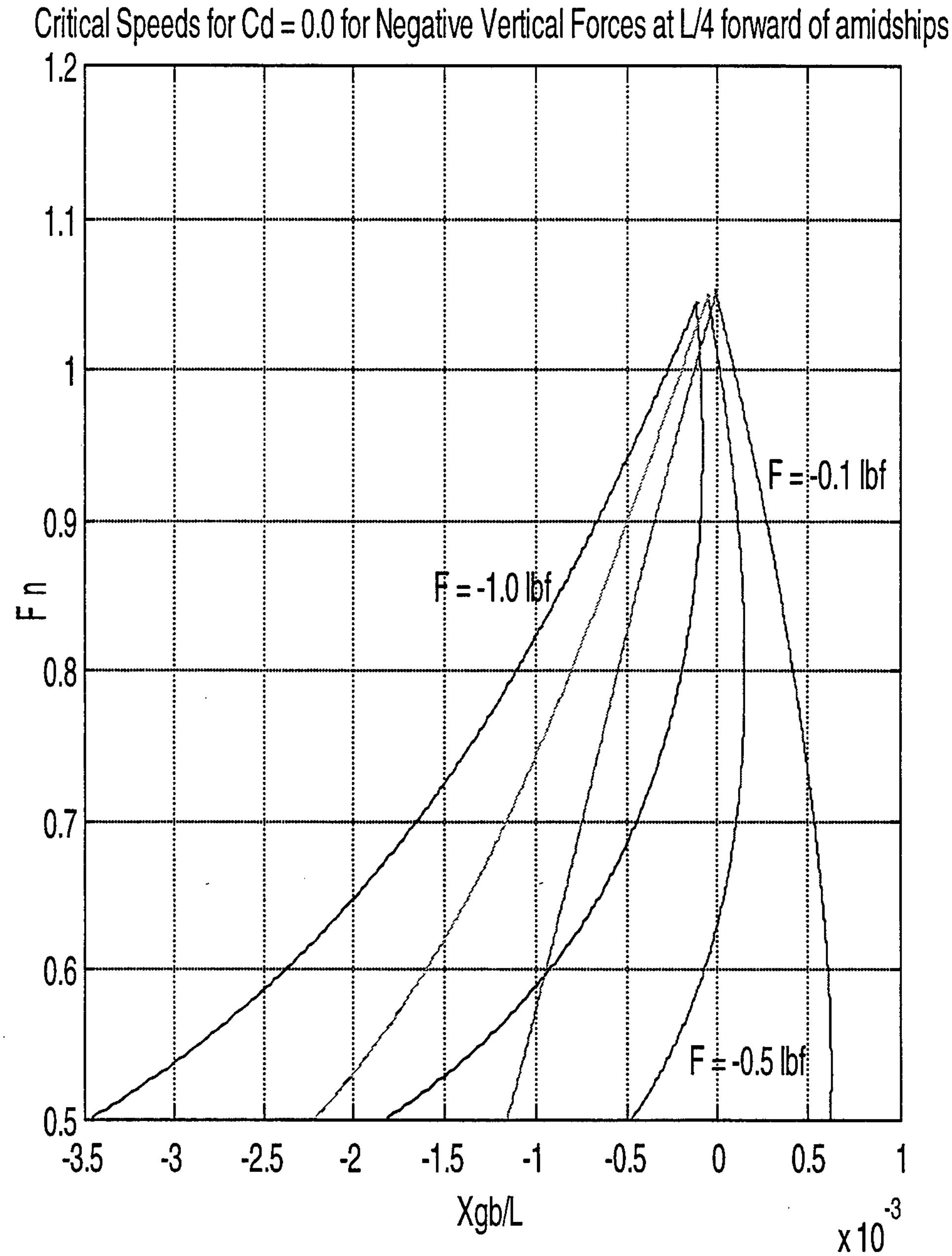

Figure 5-11 Critical Speed Envelopes Subject to Various Negative Moments at $C_D = 0.0$

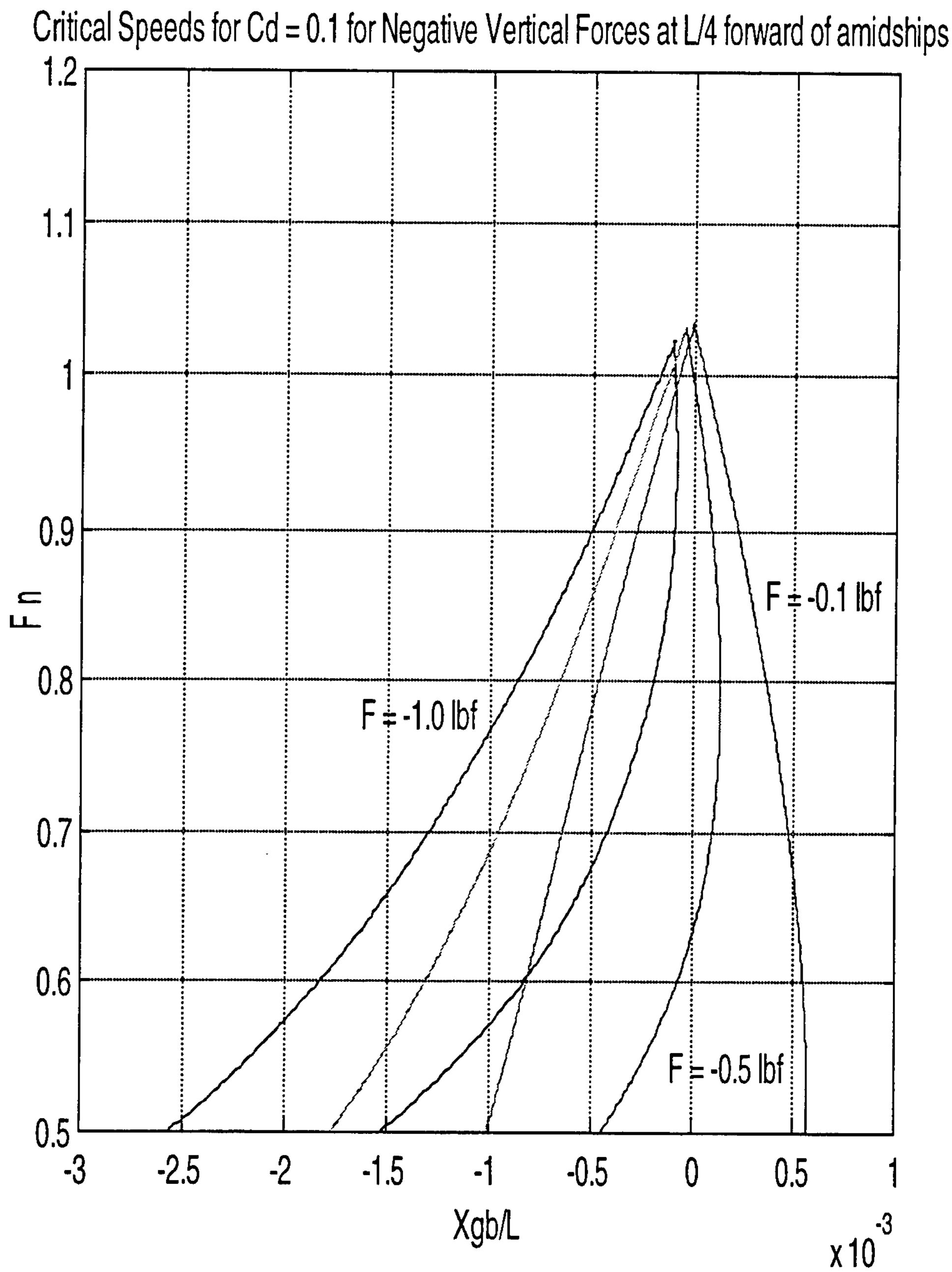

Figure 5-12 Critical Speed Envelopes Subject to Various Negative Moments at $C_D = 0.1$

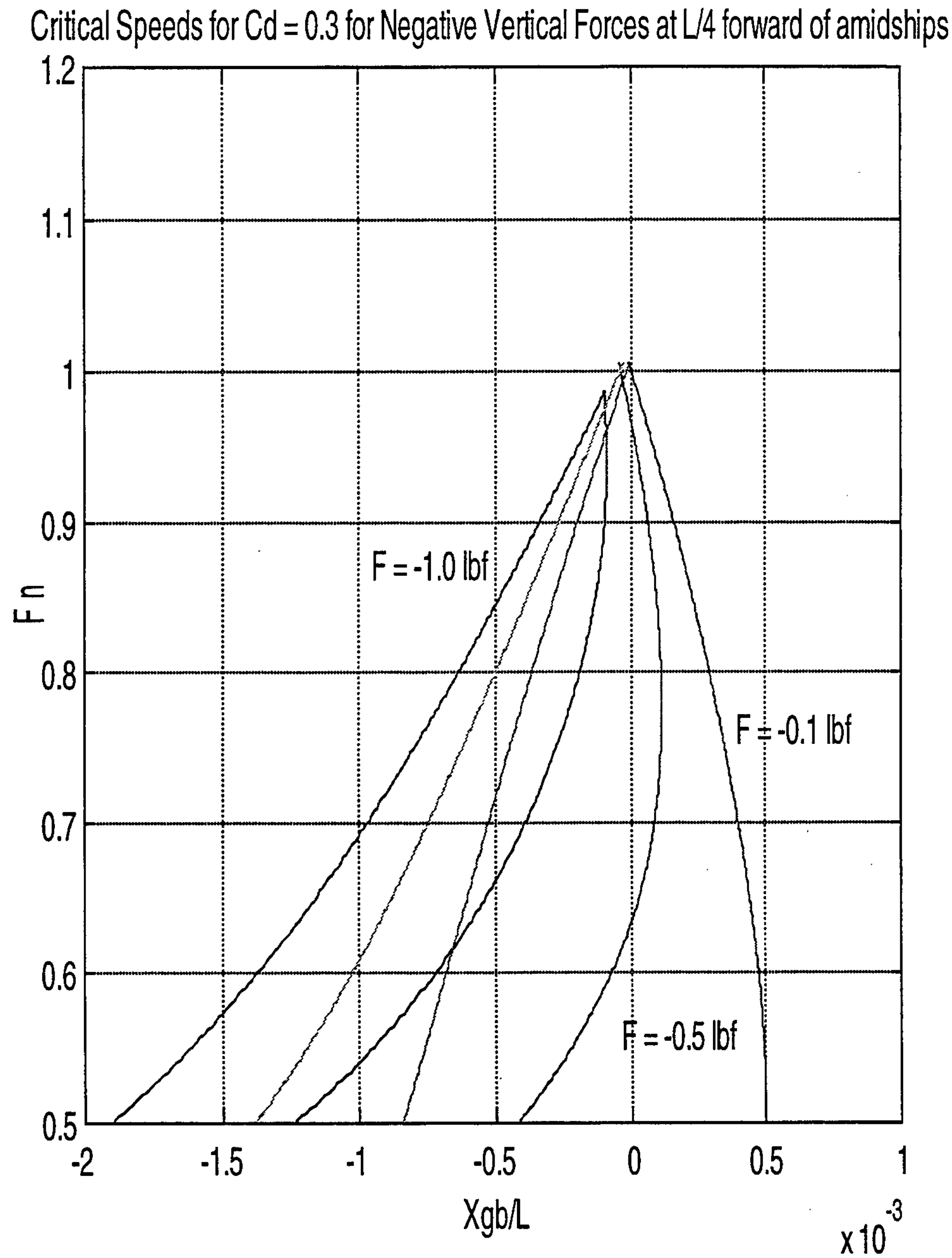

Figure 5-13 Critical Speed Envelopes Subject to Various Negative Vertical Forces at $C_D = 0.3$

VI. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The main conclusions of this work can be summarized as follows:

1) Positive vertical forces tend to lower a submersible's critical speed while shifting the critical speed envelope to the right on the Xgb/L axis.
2) Negative vertical forces tend to lower a submersible's critical speed while shifting the critical speed envelope to the left on the Xgb/L axis.
3) Positive and negative moments tend to lower a submersible's critical speed. The critical speed envelope shift is similar to that of positive and negative external forces. However, the shift is less pronounced.

The most notable feature of all results that were generated in this work is the extreme sensitivity of the parameter envelopes to the magnitude of the external forces on the vehicle. The regions where dive plane reversal is predicted to occur seem to shift dramatically even under the smallest external excitation. Although such a shift can be potentially very difficult to deal with in an automatic control system, it can be utilized to the designer's advantage since, in principle, it could establish innovative ways of accurate determination of external forces and moments. Such an identification scheme could prove very useful in shallow water mine hunting and other similar operations in littoral waters or the surf zone. Such advanced disturbance observation schemes along with robust control studies are recommendations for further research.

B. RECOMMENDATIONS

Utilize the findings of this thesis in the development of robust control systems for unmanned submersible vehicles that are operating at low speeds in shallow waters. Such control systems must incorporate sufficient fault tolerance properties so that they can accurately detect an envelope crossing and adjust the control settings accordingly.

APPENDIX

COMPUTER PROGRAMS

```
%THIS IS PROGRAM FORCER.M.  IT COMPUTES THE BIASED
BIFURCATION CUSP FOR
%DELTA SATURATED INCLUDING DRAG TERMS.  IT USES THE FZERO
FUNCTION TO
%SOLVE THE EQUATIONS.

global W B1 U n Zw Zdlt A XA CD rho F xf;

%ESTABLISH GEOMETRIC PARAMETERS
W = 1556.2363;
percent=input('Enter percent of excess buoyancy ');
B1 = (1+percent/100)*W;
F  = input('Enter vertical force ');
xf = input('Enter distance from amidships where force
acts');
A = 19.8473;
XA = 0.20126;
CD=input('Enter CD ');
Iy = 561.32;
g = 32.2;
m = W/g;
rho = 1.94;
L = 13.9792;
xb = 0;

Alphas = +1;
Alphab = 0;
zg = 0.1;
zb = 0;
zgb = zg-zb;

%NON-DIMENSIONING FACTORS

nd1 = 0.5*rho*L^2;
nd2 = 0.5*rho*L^3;
nd3 = 0.5*rho*L^4;
nd4 = 0.5*rho*L^5;

%HYDRODYNAMIC COEFFICIENTS
Zqdnd = -6.33e-4;
Zwdnd = -1.4529e-2;
Zqnd = 7.545e-3;
Zwnd = -1.391e-2;
```

```
Zds = -5.603e-3;
Zdb = 0.5*Zds;
Zdltnd = (Alphas*Zds+Alphab*Zdb);
Mqdnd = -8.8e-4;
Mwdnd= -5.61e-4;
Mqnd = -3.702e-3;
Mwnd = 1.0324e-2;
Mds = -0.002409;
Mdb = 0.5*(-Mds);
Mdltnd = (Alphas*Mds+Alphab*Mdb);

Zqd = nd3*Zqdnd;
Zwd = nd2*Zwdnd;
Zq = nd2*Zqnd;
Zw = nd1*Zwnd;
Zdlt = nd1*Zdltnd;

Mqd = nd4*Mqdnd;
Mwd = nd3*Mwdnd;
Mq = nd3*Mqnd;
Mw = nd2*Mwnd;
Mdlt = nd2* Mdltnd;

%ESTABLISH SPEED RANGE

U = 0.9:0.005:2.6;
Fn = U./sqrt(g.*zg);
for n=1:length(U);

%SOLVES FORCE EQUATION USING FZERO FUNCTION
th(n)= fzero('gsatur',(-Zdlt/Zw),0.00001);
th1(n)= fzero('gsatur1',(-Zdlt/Zw),0.00001);
%COMPUTES RELATIONSHIP BETWEEN U AND Xgb WITH RESULTS FROM
ABOVE EQN

xg(n) = (Mw*U(n)^2*tan(th(n))-zg*W*sin(th(n))-
(0.5)*rho*CD*A*XA*U(n)^2*...
tan(th(n))*abs(tan(th(n)))+Mdlt*U(n)^2*(0.4)-
F*xf)/(W*cos(th(n)));
end;

XgbL = xg/L;
Lp = sqrt(U.^2/3.22);

for n=1:length(U);

xg1(n) = (Mw*U(n)^2*tan(th1(n))-zg*W*sin(th1(n))-
(0.5)*rho*CD*A*XA*...
```

```
   U(n)^2*tan(th1(n))*abs(tan(th1(n)))+Mdlt*U(n)^2*(-0.4)-
F*xf)/...
   (W*cos(th1(n)));
end;

XgbL1= xg1/L;
plot(XgbL,Fn,XgbL1,Fn),grid
title('Saturation Cusp ');
xlabel('Xgb');
ylabel('Fn');



%THIS IS PROGRAM GSATUR.M.  IT ESTABLISHES THE FUNCTION TO
BE SOLVED IN
%THE BIFSU.M PROGRAM.

function y = gsatur(th);
global W B1 U n Zw Zdlt A XA CD rho F xf;
y= Zw*U(n)^2*tan(th)+(W-B1)*cos(th) + Zdlt*U(n)^2*(0.4)...
-(0.5)*rho*CD*A*U(n)^2*tan(th)*abs(tan(th)) - F;


%THIS IS PROGRAM GSATUR1.M.  IT ESTABLISHES THE FUNCTION TO
BE SOLVED IN
%THE BIFSU.M PROGRAM.

function y1 = gsatur1(th1);
global W B1 U n Zw Zdlt A XA CD rho F xf;
y1 =Zw*U(n)^2*tan(th1)+(W-B1)*cos(th1) + Zdlt*U(n)^2*(-
0.4)...
-(0.5)*rho*CD*A*U(n)^2*tan(th1)*abs(tan(th1)) - F;
```

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3. Department of the Navy Technical Manual 0911-003-6010, *Fundamentals of Submarine Hydrodynamics, Motion and Control,* 1971

4. David Taylor Research Center Report 2510, *Standard Equation for Submarine Simulations,* by M. Gertler and G. R. Hagen, 1967

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